Physics: Kinematics in Two-Dimensions
Class Examples

Sections Covered

- Chapter 3, Sections 4-6: pages 33 - 39

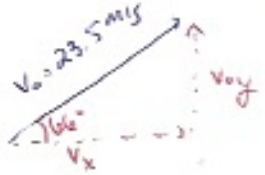
Topics Covered

- Motion in two dimensions (x and y)
- Projectile Motion
  - ½Projectile
  - Full projectile

Full Projectile

Example 1:
A football is kicked with an initial velocity (v_o) of 23.5 m/s at an angle of 66.0° above the horizontal. Find the following:

A. The maximum height
B. The hang time or time of flight
C. The range
D. At the moment of impact, the velocity in the x-direction, velocity in the y-direction, the magnitude of the final velocity, and the angle of impact.



$v_x = \cos(66.0°) \cdot 23.5$
$v_x = 9.56$ m/s
$v_{oy} = \sin(66.0°) \cdot 23.5$
$v_{oy} = 21.5$ m/s

x
x = 42.0 m
v_x = 9.56 m/s
t = 4.39 s (to the end)

y
y = 0 m (start and end)
y_{max} = 23.6 m
v_{oy} = 21.5 m/s
v_y =
a_y = -9.8 m/s²
t = 4.39 s (to the end)

A) max Height - the y-velocity will be 0 m/s at this point
y_{max} = ?
v_{oy} = 21.5 m/s
v_y = 0 m/s
a_y = -9.8 m/s²

$v_y^2 = v_{oy}^2 + 2a_y \cdot y_{max}$
$(0)^2 = (21.5)^2 + 2(-9.8) y_{max}$
$0 = 462.25 - 19.6 \cdot y_{max}$
$-462.25 = -19.6 \cdot y_{max}$
$y_{max} = 23.6$ m

B.) Hang time - time when the ball is back to a position of y = 0 m
t = ?
y = 0 m
v_{oy} = 21.5 m/s
a_y = -9.8 m/s²

$y = v_{oy}t + \frac{1}{2}a_y t^2$
$0 = 21.5 \cdot t + \frac{1}{2}(-9.8) t^2$
$0 = 21.5 \cdot t - 4.9 t^2$
$0 = (21.5 - 4.9t)\, t$
two solutions - set both = 0
t = 0 s

$21.5 - 4.9t = 0$
$-4.9t = -21.5$
$t = \frac{-21.5}{-4.9}$
t = 4.39 s

C.) Range
$x = v_x \cdot t$
$x = (9.56)(4.39)$
x = 42.0 m

Example #1 → Part D

Final Velocity

X-velocity is constant

$v_x = 9.56$ m/s

y-velocity changes

v_{fy} = ?

$v_{oy} = 21.5$ m/s

$a_y = -9.8$ m/s²

$t = 4.39$ s

$v_{fy} = v_{oy} + at$

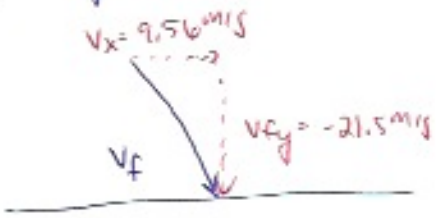
$v_{fy} = 21.5 + (-9.8)(4.39)$

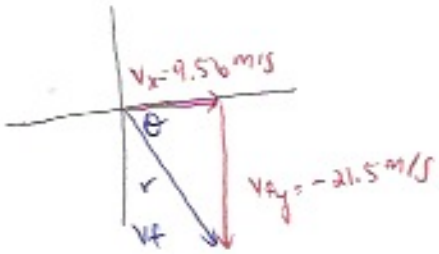
$v_{fy} = 21.5 + (-43.0)$

$v_{fy} = -21.5$ m/s

Rectangular Coordinates of the final velocity (9.56, -21.5)

turn into polar (magnitude, angle)





$r^2 = (9.56)^2 + (-21.5)^2$

$r^2 = 91.3936 + 462.25$

$r^2 = 553.644$

$r = \sqrt{553.644}$

$r = 23.5$ m/s

$\theta = \tan^{-1}\left(\frac{-21.5}{9.56}\right)$

$\theta = -66.0°$

Initial Velocity

23.5 m/s at 66.0°

Final Velocity

23.5 m/s at -66.0°

*Symmetry