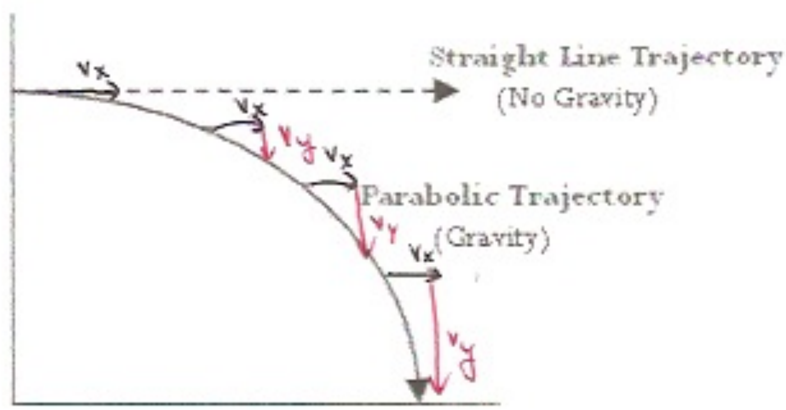
Honors Physics:
Problem Set – Two-Dimensional Kinematics
**Set calculator to degrees*

Conceptual Ideas

1. What are things that can ALWAYS be assumed with projectile motion? • Time is the same whether measured in the x- or y-direction
   - The only acceleration is gravity $(-9.8\ m/s^2)$ which affects variables in the y-direction.
   - Therefore, acceleration in the x-direction is ALWAYS zero. $a_x = 0\ m/s^2$
   - With no acceleration, velocity in the x-direction is CONSTANT $v_{ox} = v_{fy} = v_x$
2. For the ½ Projectile shown below, please answer the following
   A. What kind of velocity does a ½ projectile have at the beginning of its motion?

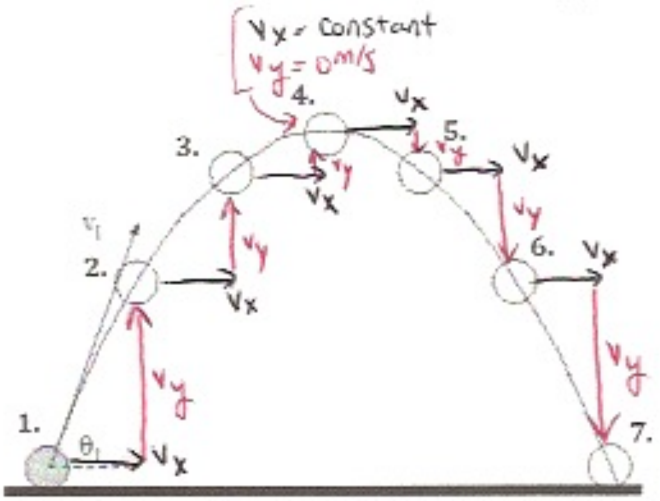
   B. What happens to the velocity in the y-direction as the objects moves?

A.) At the beginning of the projectile's motion, the y-velocity (v_{oy}) is zero.
$v_{oy} = 0\ m/s$
Therefore, the only velocity initially is the horizontal, x-velocity (v_x)

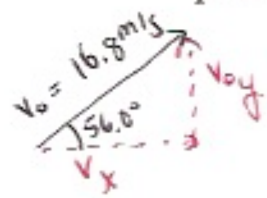


B.) As the projectile moves, the velocity in the y-direction increases.
The velocity in the x-direction remains constant.

3. The full projectile below has an initial velocity of 16.8 m/s directed at an angle of 56.0°.



A. What are the values of the velocity in the x- and y- direction at the initial point (point #1)



$\cos(56.0°) = \frac{V_x}{16.8}$

$V_x = \cos(56.0°) \cdot 16.8$

$\boxed{V_x = 9.39 \text{ m/s}}$

$\sin(56.0°) = \frac{V_{oy}}{16.8}$

$V_{oy} = \sin(56.0°) \cdot 16.8$

$\boxed{V_{oy} = 13.9 \text{ m/s}}$

B. What are the values of the velocity in the x- and y- direction at the highest point (point #4)

At the highest point, velocity in the y-direction is zero.

But, velocity in the x-direction remains constant

At the highest point: $\boxed{V_x = 9.39 \text{ m/s} \quad V_y = 0 \text{ m/s}}$

C. What are the values of the velocity in the x- and y- direction at the point of impact (point #7)

Because the projectile is symmetric, the velocities are the same in magnitude as the initial values - but the y-velocity has a negative sign to indicate a downward direction at the point of impact.

$\boxed{V_x = 9.39 \text{ m/s} \quad V_{fy} = -13.9 \text{ m/s}}$

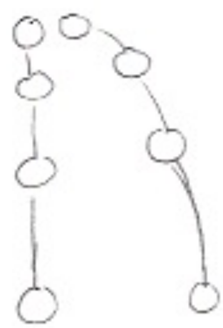
D. What happens to the x-velocity throughout the course of the projectile's movement?

The velocity in the x-direction remains constant

E. What happens to the y-velocity throughout the course of the projectile's movement?

The velocity in the y-direction decreases during the ascent, is 0 m/s at the highest point, then becomes increasingly negative on the way down

4. In one of our very first demonstrations with projectile motion, an apparatus was used to launch one ball out as a projectile, while allowing another to fall from the exact same height. Which hit the ground first? Why?



They will hit the ground at the same time. Gravity affects them both equally despite any horizontal velocity.

5. How does the launch angle affect the trajectory of a projectile?

Smaller Launch Angles

more shallow trajectory
emphasizes horizontal motion

Larger launch Angle

much steeper trajectory
emphasizes vertical motion

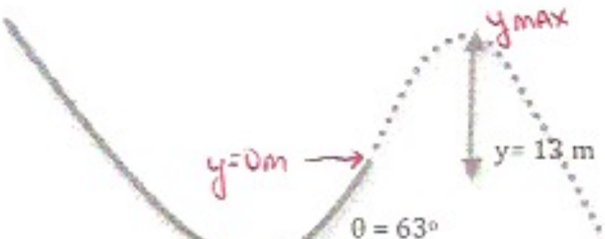
In the women's Olympic ski jump competition, the end of a typical ramp is angled 63.0° above the horizontal. With this launch angle, a skier attains a maximum height of 13.0 m above the end of the ramp.

Projectile Type: FULL PROJECTILE

A. What is the initial velocity in the y-direction?

B. How long does it take the skier to reach this maximum height?

C. What is the value of the initial velocity (v_o)?



x

x =

v_x =

t =

y

y = 0 m at the beginning

y_{MAX} = 13.0 m

V_{oy} - from part A: 16.0 m/s

V_{fy} =

$a_y = -9.8\ m/s^2$

t =

A.) V_{oy} = ?

$V_{fy} = 0\ m/s$

$a_y = -9.8\ m/s^2$

$y_{MAX} = 13.0\ m$

* We have information about the maximum y-value. What we also know is that at the max. y-value, the velocity in the y-direction is zero. So $V_{fy} = 0\ m/s$

$$V_{fy}^2 = V_{oy}^2 + 2a_y y$$

$$(0)^2 = V_{oy}^2 + 2(-9.8)(13)$$

$$0 = V_{oy}^2 - 254.8$$

$$254.8 = V_{oy}^2$$

$$V_{oy} = \pm 15.9625$$

$$\boxed{V_{oy} = 16.0\ m/s}$$

B.) $V_{oy} = 16.0\ m/s$

$V_{fy} = 0\ m/s$

$y = 13.0\ m/s$

$a_y = -9.8\ m/s^2$

t =

$$V_{fy} = V_{oy} + a_y t$$

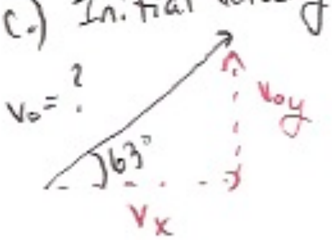
$$0 = 16.0 + (-9.8)t$$

$$-16.0 = -9.8t$$

$$t = \frac{-16.0}{-9.8}$$

$$\boxed{t = 1.63\ s}$$

C.) Initial Velocity (v_o)



$$V_x = \cos(63.0°) \cdot V_o$$

$$V_{oy} = \sin(63.0°) \cdot V_o \longrightarrow 16.0 = \sin(63°) \cdot V_o$$

* We know V_{oy}

$V_{oy} = 16.0\ m/s$

$$V_o = \frac{16.0}{\sin(63°)}$$

$$\boxed{V_o = 18.0\ m/s}$$

A projectile is launched with an initial velocity of 75.2 m/s at an angle of 34.5° above the horizontal. Find the following.

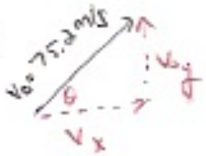
Projectile Type: FULL PROJECTILE

A. What are the x- and y- components of the initial velocity?

B. What are the x- and y-components of the velocity of the projectile at t = 0.750 s? Find the magnitude and angle (θ) of the resultant velocity at t = 0.750 s

x	y
$x =$	$y = 0$ m Start/end
$v_x = 62.0$ m/s	$y_0 =$
$t =$	$v_{0y} = 42.6$ m/s
	$v_{fy} =$
	$a_y = -9.8$ m/s²
	$t =$

A.)



$v_x = \cos(34.5°) \cdot 75.2$

$\boxed{v_x = 62.0 \text{ m/s}}$

$v_{0y} = \sin(34.5°) \cdot 75.2$

$\boxed{v_{0y} = 42.6 \text{ m/s}}$

B.) x- and y- components of velocity at t = 0.750 s.

* projectile still on its way up

→ we'll use v_{fy} as the variable in our kinematics

$\boxed{v_x = 62.0 \text{ m/s because } v_x \text{ is constant}}$

$v_{fy} = ?$

$v_{0y} = 42.6$ m/s

$a_y = -9.8$ m/s²

$t = 0.750$ s

$v_{fy} = v_{0y} + a_y \cdot t$

$v_{fy} = 42.6 + (-9.8)(0.750)$

$v_{fy} = 42.6 - 7.35$

$\boxed{v_{fy} = 35.3 \text{ m/s}}$

What is the resultant and angle of these two components

$v_{fy} = 35.3$ m/s

$v_x = 62.0$ m/s

$\theta = \tan^{-1}\left(\frac{35.3}{62.0}\right)$

$\boxed{\theta = 30.0°}$

$v^2 = v_x^2 + v_{fy}^2$

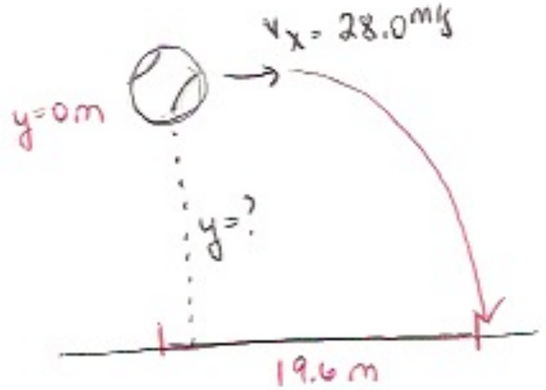
$v^2 = (62.0)^2 + (35.3)^2$

$v^2 = 3844 + 1246.09$

$\boxed{v = 71.3 \text{ m/s}}$

A tennis ball is slammed with an initial horizontal velocity of 28.0 m/s. The ball hits the tennis court at a horizontal distance of 19.6 m from the racket. What is the height of the tennis ball above the court when it leaves the racket?

Projectile Type: ½ PROJECTILE



x

$x = 19.6$ m

$v_x = 28.0$ m/s

$t =$

y

$y = 0$ m at the top

$y =$ at the bottom

$v_{oy} = 0$ m/s

$v_{fy} =$

$a_y = -9.8$ m/s²

$t =$

* We want to find the value of y when the ball reaches the ground, to know how far it fell. BUT we need one more variable. So we can go to our x-variables. From here, we can solve for time.

$x = v_x t$

$19.6 = 28.0 \cdot t$

$t = \frac{19.6}{28.0}$

$\boxed{t = 0.700\ s}$

$y =$

$a_y = -9.8$ m/s²

$v_{oy} = 0$ m/s

from our x-variables

$t = 0.700$ s

$y = v_{oy} t + \frac{1}{2} a_y t^2$

$y = (0) \cdot (0.700) + \frac{1}{2}(-9.8)(0.700)^2$

$y = 0 - (4.9)(0.49)$

$y = -2.40$ m

So the ball fell 2.40 m to the ground
Height above the ground = 2.40 m

A diver runs 1.60 m/s horizontally off the edge of the diving board. He reaches the water below in 3.00 s.

Projectile Type: ½ PROJECTILE

A. How high was the Diving Board.

B. How far from the base of the diving board did he hit the water?

$V_x = 1.60\,m/s$

<u>x</u>

x =

$V_x = 1.60\,m/s$

t = 3.00 s

<u>y</u>

y = 0 m at the top

y = at the bottom

$V_{oy} = 0\,m/s$

$V_{fy} =$

$a_y = -9.8\,m/s^2$

t = 3.00 s

A.) * Similar to Problem # 7, we want to find the value of y at the bottom, when the diver hits the water

y =

$V_{oy} = 0\,m/s$

$a_y = -9.8\,m/s^2$

t = 3.00 s

$y = V_{oy}t + \frac{1}{2}a_y t^2$

$y = (0)(3.00) + \frac{1}{2}(-9.8)(3.00)^2$

$y = 0 - 44.1$

$y = -44.1\,m$

Diving Board was 44.1m in the air (pretty high!)

B.) $x = V_x t$

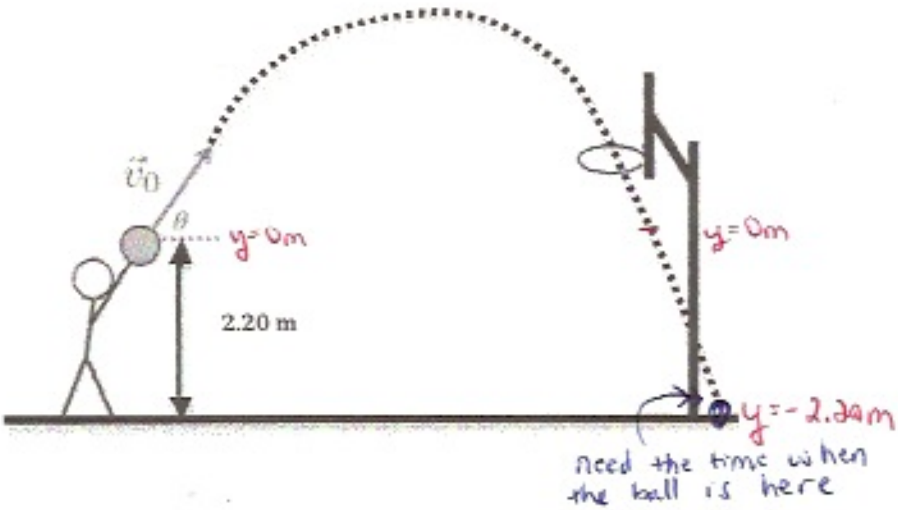
$x = (1.60)(3.00)$

$\boxed{x = 4.8\,m}$

*A basketball player shoots the ball with an initial velocity (v_o) = 14.0 m/s at an angle of 40.0° with respect to the ground.

Projectile Type: FULL PROJECTILE

What is the range or horizontal distance the ball travels if it is released 2.20 m above the ground?



X

X=

$V_x = 10.7$ m/s

t=

y

y= 0 m at the beginning

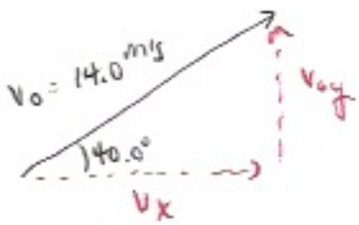
y= -2.20m at the floor

V_{oy}= 9.00 m/s

V_{fy}=

a_y= -9.8 m/s²

t=



$V_x = \cos(40.0°) \cdot 14.0$

$V_x = 10.7$ m/s

$V_{oy} = \sin(40.0°) \cdot 14.0$

$V_{oy} = 9.00$ m/s

* To find the Range (x), we will need to find time (t)

$x = V_x t$

$x = (10.7)\, t$ *using time found below t=2.05 s

$x = (10.7)(2.05)$

$\boxed{x = 21.9 \text{ m}}$

*To find time, we will need to use our y-variables. when looking for the range. we need the time when the ball is on the ground

t=

y= -2.20 m

V_{oy}= 9.00 m/s

a_y= -9.8 m/s²

$y = V_{oy} t + \frac{1}{2} a_y t^2$

$-2.20 = 9.00\, t + \frac{1}{2}(-9.8) t^2$

$-2.20 = 9.00\, t - 4.9 t^2$

QUADRATIC: RE-ARRANGE

$4.9 t^2 - 9.00\, t - 2.20 = 0$

Because time cannot be negative

$\boxed{t = 2.05 \text{ s}}$

~~t = -0.214 s~~

QUADRATIC FORMULA

$t = \dfrac{9.00 \pm \sqrt{(-9.00)^2 - 4(4.9)(-2.20)}}{2(4.9)}$

$t = \dfrac{9.00 \pm \sqrt{81 + 43.12}}{9.8}$

$t = \dfrac{9.00 \pm \sqrt{124.12}}{9.8}$

$t = \dfrac{9.00 \pm 11.1}{9.8}$

$t = \dfrac{9.00 + 11.1}{9.8} = 2.05 \text{ s}$ $\quad t = \dfrac{9.00 - 11.1}{9.8} = -0.214$