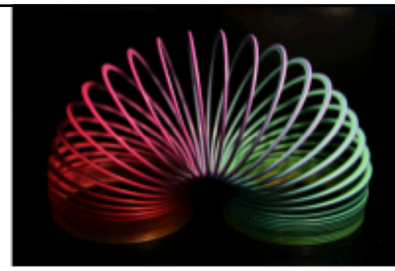
Rotational Formulas



- We will be creating three **time-dependent** functions.
  - One for position (x)
  - One for velocity (v)
  - One for acceleration (a)

$x = A \cdot \cos\theta$

$x = A \cdot \cos(\omega t)$

$x_{MAX} = A$

$\omega = \frac{\Delta\theta}{\Delta t}$

$\omega = \frac{\theta}{t}$

$\theta = \omega \cdot t$

$v = -A \cdot \omega \sin(\omega t)$ 1 or -1

$v_{MAX} = +/- A \cdot \omega$

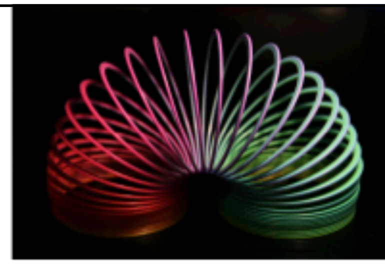
$a = -A\omega\cos(\omega t) \cdot \omega$

$a = -A\omega^2\cos(\omega t)$

$a = -A\omega^2\cos(\omega t)$

$a_{MAX} = +/- A\omega^2$

Example 4



A 2.00 kg mass oscillates back and forth on a spring with a period of 0.550 s. The mass was stretched 0.100 m and released to begin its motion.

A. Write a time-dependent function for the position of this spring-mass system.

B. Where is this system at t = 2.00 s?

C. Write a time-dependent function for the velocity of this spring-mass system.

D. What is the velocity this system at t = 2.00 s?

E. What is the maximum velocity of this system?

$m = 2.00\,kg$

$T = 0.55\ s$

$A = 0.100\ m$

A.) $x = A\cos(\omega \cdot t)$

$x = 0.1\cos(11.4 \cdot t)$

$x = 0.1\cos(11.4 \cdot 2) = -0.07\ m$

$\omega = \frac{2\pi}{T} = 11.4\ rad/s$

$v = -A \cdot \omega \sin(\omega t)$

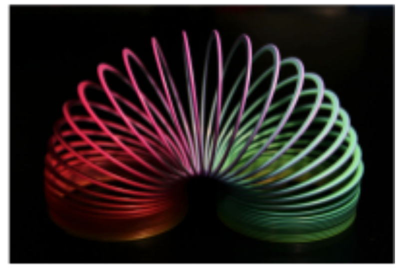
$v = -(0.1)(11.4)\sin(11.4t)$

$v = -1.14\sin(11.4 \cdot t)$

$v = 0.86\ m/s$

$v_{MAX} = \pm 1.14\ m/s$

Important Formulas - Derived



- Start with Hooke's Law

$F = -kx = ma$

$-kx = ma$

$\cancel{-}k \cdot \cancel{A}\cancel{\cos(\omega t)} = m(\cancel{-}\cancel{A}\omega^2 \cancel{\cos(\omega t)})$

$k = m\omega^2$

$\frac{k}{m} = \omega^2$

$\sqrt{\frac{k}{m}} = \omega$

- Formulas for x, v, and a

$$x = A\cos(\omega t)$$

$$v = -A\cdot \omega \sin(\omega t)$$

$$a = -A\cdot \omega^2 \cos(\omega t)$$

$\omega = \sqrt{\frac{k}{m}}$

Important Formulas - Derived

$$\omega = \sqrt{\frac{k}{m}} \qquad \omega = \frac{2\pi}{T}$$

$$\sqrt{\frac{k}{m}} = \frac{2\pi}{T}$$

$$\frac{\sqrt{k}}{\sqrt{m}} = \frac{2\pi}{T}$$

$$\frac{2\pi\sqrt{m}}{\sqrt{k}} = \frac{T\sqrt{k}}{\sqrt{k}}$$

$$2\pi\sqrt{\frac{m}{k}} = T$$

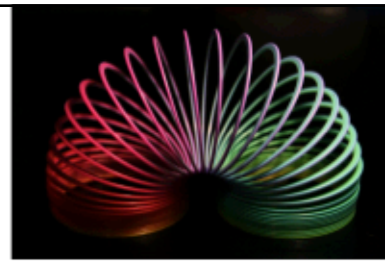
$$\frac{k}{m} = \frac{(2\pi)^2}{T^2}$$

$$m(2\pi)^2 = k \cdot T^2$$

$$\sqrt{\frac{m(2\pi)^2}{k}} = \sqrt{T^2}$$

$$2\pi\sqrt{\frac{m}{k}} = T$$

Formulas



- List all the formulas we have so far

Pendulum

$\omega = \frac{2\pi}{T}$ $\quad F_s = -kx$ $\quad T = 2\pi\sqrt{\frac{L}{g}}$

$^{*}T = 2\pi\sqrt{m/k}$

$X = A\cos(\omega t)$

$V = -A\omega\sin(\omega t)$

$V_{max} = \pm A\omega$

$a = -A\omega^2\cos(\omega t)$

$X_{max} = A$

$a_{max} = A\omega^2$

$\omega = \sqrt{\frac{k}{m}}$

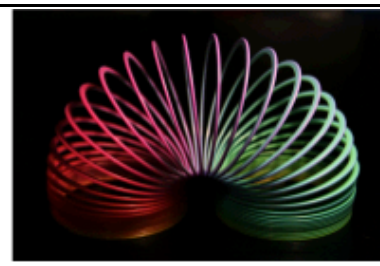
$\omega = 2\pi f$ $\quad W = \frac{\Delta\theta}{\Delta t}$

$f = \frac{1}{T}$ $\quad T = \frac{1}{f}$

$\underbrace{U_s = \frac{1}{2}kx^2 \quad KE = \frac{1}{2}mv^2}_{mgh}$

$rot = \frac{1}{2}I\omega^2$

Example 5



A block of mass M is resting on a horizontal, frictionless table and is attached as shown above to a relaxed spring of spring constant k. A second block of mass 2M and initial speed v_o collides with and sticks to the first block. Develop expressions for the following quantities in terms of M, k, and v_o.

A. v, the speed of the block immediately after impact.

B. x, the maximum distance the spring is compressed.

C. T, the period of the subsequent simple harmonic motion.

$x = \sqrt{\frac{4Mv_0^2}{3k}}$

$\frac{1}{2}mv^2 = \frac{1}{2}kx^2$

$\frac{1}{2}(3)M\left(\frac{2}{3}v_0\right)^2 = \frac{1}{2}kx^2$

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$\frac{1}{2}mv^2$

A.) Conservation of momentum $p = m \cdot v$

$p_o = p_f$

$p_{larger\ block} + p_{smaller\ block} =$

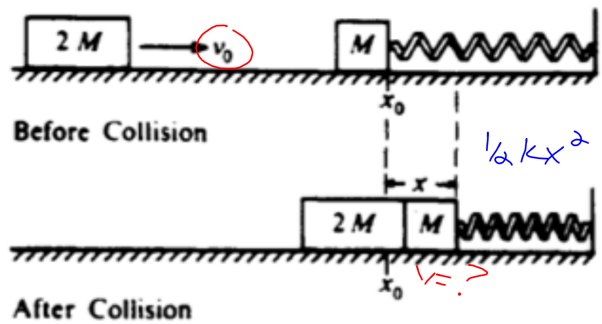
$2Mv_0 + M(\cancel{0}) = (2M+M)v$

$2Mv_0 = 3Mv$

$\frac{2}{3}v_0 = v$

$T = 2\pi\sqrt{\frac{m}{k}}$

$T = 2\pi\sqrt{\frac{3M}{k}}$



$\frac{1}{2}kx^2$

$v = ?$