

Moment of Inertia

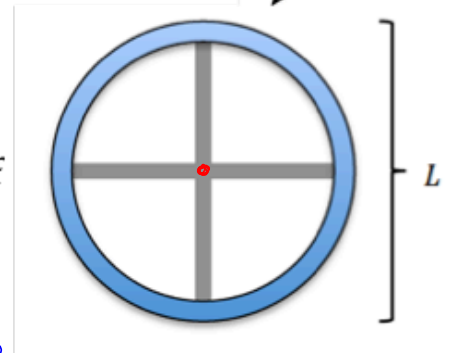
Warm-Up

Two thin rods are attached to a thin hoop to create a wheel as shown. The rods both have a mass M and a length L . The hoop also has a mass M and a radius of $L/2$. The axis of rotation is at the intersection point of the rods at the center of the hoop.

Calculate the moment of inertia for the entire system

$$\sum I = \frac{1}{12}ML^2 + \frac{1}{12}ML^2 + M\left(\frac{L}{2}\right)^2$$

$$I = \frac{5}{12}ML^2$$



Energy



- Review: What kinds of energy do we currently know of?

$$KE = \frac{1}{2} m v^2$$

Translation

$$U = mgh$$

+ thermal energy

...and what is meant by conservation of energy?

$$F = ma$$

$$\tau = I \cdot \alpha$$

$$KE = \frac{1}{2} m v^2$$

$$KE = \frac{1}{2} (m r^2) \omega^2$$

$$KE = \frac{1}{2} I \omega^2$$

$$v = r \cdot \omega$$

$$v^2 = r^2 \cdot \omega^2$$

Kinetic Energy: $KE = \frac{1}{2} m v^2$

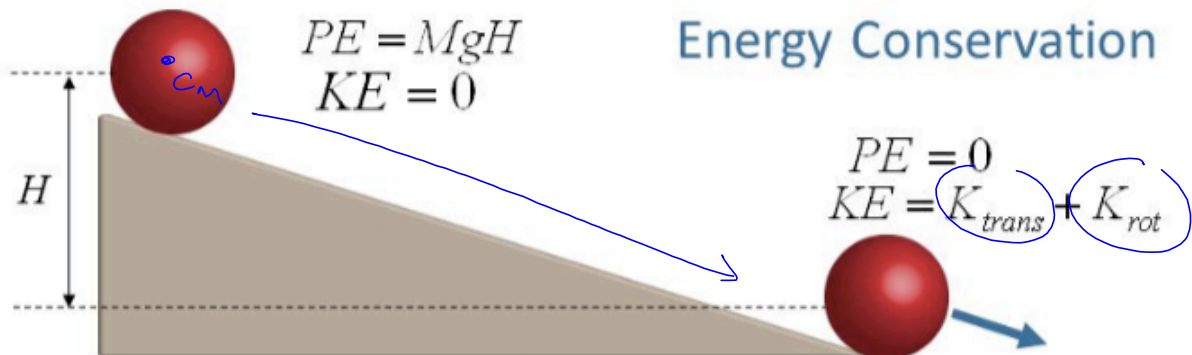
Rotational Kinetic Energy:

$$KE_{rot} = \frac{1}{2} I \omega^2$$

Translational & Rotational Energy



- Total Kinetic Energy
 - $KE = \frac{1}{2} mv^2 + \frac{1}{2} I\omega^2$
- Conservation of Energy




Example 11

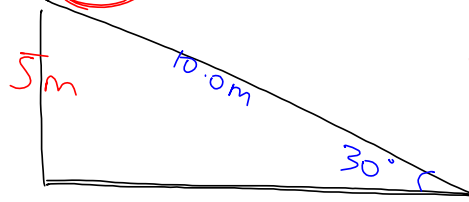


A sphere of radius 20.0 cm and a mass of 1.80 kg starts from rest and rolls without slipping down a 30.0° incline that is 10.0 m long.

A. Calculate its translational and rotational speeds when it reaches the bottom. ω v

B. Do your answers in part A depend on the radius and/or mass of the sphere?

① 



$m = 1.80 \text{ kg}$
 $r = 20 \text{ cm} = 0.20 \text{ m}$

$E_1 = E_2$
 $mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$
 $I = \frac{2}{5}mR^2$
 $\omega = \frac{v}{r}$
 $\omega = 41.85 \frac{\text{rad}}{\text{s}}$

$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{2}{5}mR^2\right)\omega^2$
 $gh = \frac{1}{2}v^2 + \frac{1}{5}r^2\omega^2$
 $gh = \frac{1}{2}v^2 + \frac{1}{5}r^2\left(\frac{v^2}{r^2}\right)$
 $gh = \frac{1}{2}v^2 + \frac{1}{5}v^2$
 $gh = \frac{7}{10}v^2$
 $v = 8.37 \text{ m/s}$

$v = r \cdot \omega$
 $\frac{v}{r} = \omega$
 $\frac{v^2}{r^2} = \omega^2$