Newton's Second Law – Rotational Motion

- The 'F = ma' for rotation is....

$F = ma$

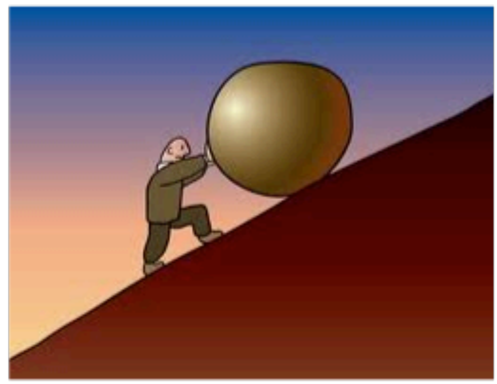
$$\tau = \underset{m}{\underset{\downarrow}{I}} \cdot \underset{a}{\underset{\downarrow}{\alpha}}$$

α Angular acceleration

I Moment of Inertia

Moment of Inertia

- Similar to mass – measure of an object's resistance to rotation



- Symbol: I
- SI Units: $kg \cdot m^2$
- * **Dependent on an object's distribution of mass**

generic

$\boxed{I = \Sigma m r^2}$ For a collection of particles or objects

The larger the moment of inertia, the more resistant an object is to changes in rotation.

Newton's Second Law – Rotational Motion Derivation

- Linear Motion:

$$\Sigma F \propto a$$

- Rotational Motion:

$$\Sigma \tau \propto \alpha$$

$$F = m a_{\perp}$$

*

$a_t = r \cdot \alpha$

$r(F) = (m\ r \cdot \alpha) r$

$r \times F = m r^2 \cdot \alpha$

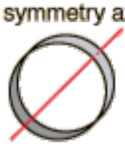
$\tau = I \cdot \alpha$

$\tau = I \cdot \alpha$ ✓

Moment of Inertia

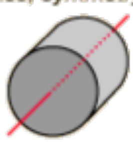
- Moment of Inertia: $I \propto mr^2$

Hoop about symmetry axis



$$I = mr^2$$

Solid cylinder or disc, symmetry axis



$$I = \frac{1}{2}mr^2$$

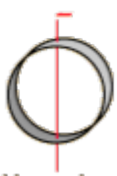
Solid sphere

$$I = \frac{2}{5}mr^2$$

Rod about center

$$I = \frac{1}{12}mL^2$$

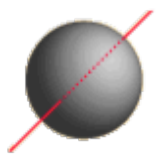
mR^2



Hoop about diameter

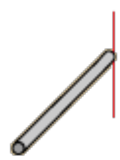
$$I = \frac{1}{2}mr^2 + \frac{1}{12}mw^2$$

*w = width of hoop



Thin spherical shell

$$I = \frac{2}{3}mr^2$$



Rod about end

$$I = \frac{1}{3}mL^2$$

*L = length of rod

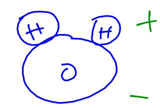
Questions

- Why is it called the 'moment' of inertia?
  - In physics, a 'moment' is a combination of a physical quantity with a distance – in this case, mass and radius (distance)

* dipole moment

*

- *Derivations....using integral via calculus
  – Good site through Mini Physics



Example 8

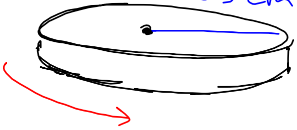
A grinding wheel is uniform cylinder with a radius of 8.50 cm and a mass of 0.580 kg. Calculate the following.

A. The moment of inertia about the cylinder's center

B. The applied torque needed to accelerate the wheel from rest to 1500 rpms in 5.00 s if it is known to slow down from 1500 rpms to rest in 55.0 s.

$m = 0.580\ kg$

8.5cm = 0.085 m



A.) $I = \frac{1}{2} m R^2$

$I = 2.10 \times 10^{-3}\ kg \cdot m^2$

B.) $\tau = ?$

$* \ \tau = I \cdot \alpha$

$\tau = r \times F \sin\theta$

$\omega_0 = 0\ rad/s \rightarrow 1500\ rpms$

$t = 5.00\ s$

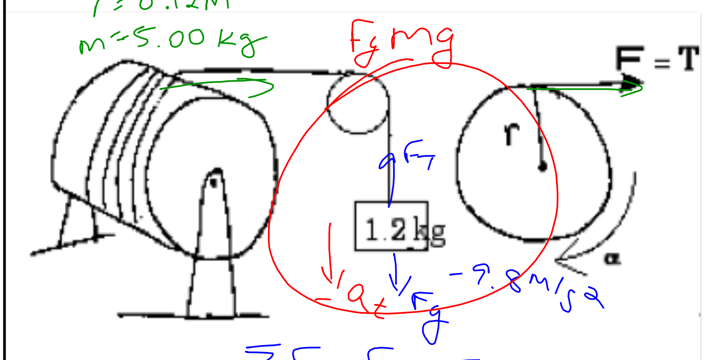
$$\frac{1500\ rev}{1\ min} \cdot \frac{2\pi\ rad}{1\ rev} \cdot \frac{1\ min}{60\ s} = 157\ rad/s$$

Example 9

A solid, uniform cylinder of 12 cm radius with mass of 5.0 kg is free to rotate about its symmetry axis. A cord is wound around the drum and a 1.2 kg mass is attached to the end of the cord. Find the acceleration of the hanging mass, the angular acceleration of the cylinder, and the tension in the cord.

$r = 0.12\,m$

$m = 5.00\,kg$



$\Sigma F_y = F_T - F_g$

$m \cdot a_t = F_T - F_g$

$-ma_t = F_T - mg$

$* \; mg - ma_t = F_T$

$a_t = r \cdot \alpha$

$a_t = (0.12)(26.4)$

$\tau = r F \sin\theta$

$\tau = (0.12) F_T \sin 90°$

$\tau = 0.12\, F_T$

$\tau = I \cdot \alpha$

$\tau = \frac{1}{2}(5.00)(0.12)^2 \cdot \alpha$

$\tau = 0.036 \cdot \alpha$

$\alpha \approx 26.5\, rad/s^2$

$a_t = -3.18\, m/s^2$

$F_T = 7.94\, N$

Example 10

A rigid bar with a mass M and length L is free to rotate about a frictionless hinge on a wall. The bar has a moment of inertia of I = 1/3ML^2 around the hinge and is released from rest from an initially horizontal position as shown. What is the angular acceleration of the bar when it makes a 30.0° angle to the vertical also as shown?

A.) $\frac{g}{3L}$

B.) $\frac{2g}{3L}$

C.) $\frac{g}{L}$

D.) $\frac{3g}{4L}$

E.) $\frac{3g}{2L}$

α

$\tau = I \cdot \alpha \qquad \tau = r \times F$

$I \cdot \alpha = r \times F \sin\theta$

$\alpha = \frac{r \times F \sin\theta}{I}$

$\alpha = \frac{\frac{L}{2} \cdot F_g \cdot \sin 30°}{1/3\, M L^2}$

$\alpha = \frac{\frac{L}{2} \cancel{M} g \sin 30°}{1/3\, \cancel{M} L^2} \longrightarrow \alpha = \frac{3 \cancel{L} g \cdot \sin\theta}{2 L^{\cancel{2}}} = \frac{3 g \sin 30°}{2L}$