Rotation

- Angular variables and rotational kinematics

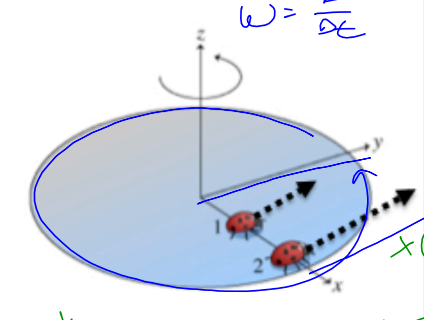
Displacement	Velocity	Acceleration
x (meters)	v (m/s)	a (m/s^2)
θ (radians)	ω (rad/s)	α (rad/s^2)

$$\omega = \omega_o + \alpha t$$

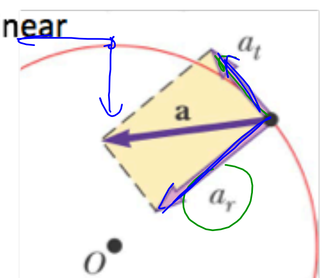
$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha t^2$$

$v^2 = v_0^2 + 2a(x - x_0)$ $\quad x = \frac{1}{2}(v_0 + v)t$

$\omega = \frac{\Delta\theta}{\Delta t}$



- All points on the wheel - same angular velocity (ω) and angular acceleration (α)
- Different linear velocity and linear acceleration (tangential)
- (as if they were to fly off)
  - faster, farther out



$$a_t = r\alpha$$

$a_r =$

$a_c = \frac{v^2}{r} = \frac{r^2\omega^2}{r}$

$$v = r\omega$$

$$a_r = \frac{v^2}{r} = r\omega^2$$

$$a = \sqrt{a_r^2 + a_t^2}$$

Rotation

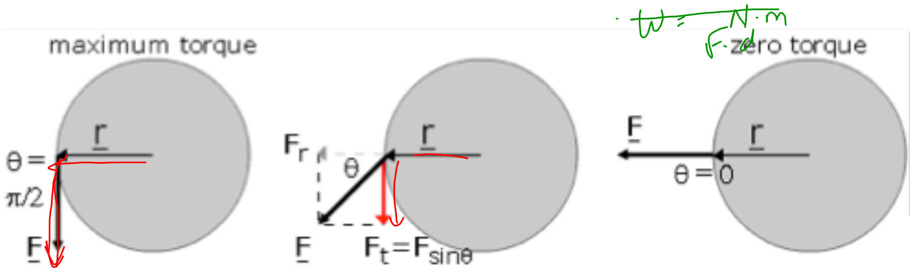
- Torque – Turning power, similar to Force (F)

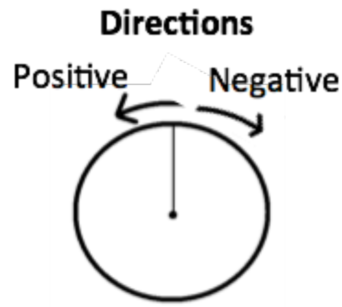
$$\tau = r \perp F = rF\sin\theta$$

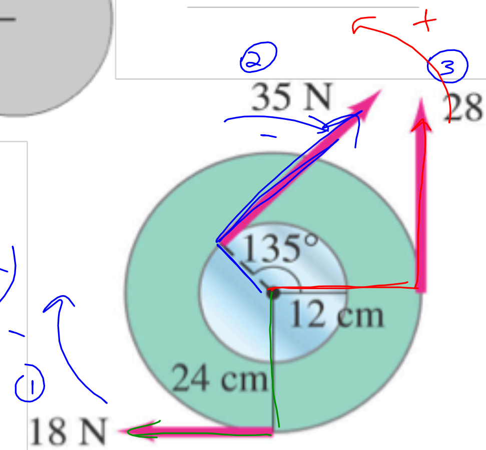
- Unit: m N

$W = N \cdot m$

$F \cdot d$







$F = m\,a$

$$\tau = I \cdot \alpha$$

$\Sigma T = T_3 - T_1 - T_2$

$\Sigma T = (0.24)(28) - (0.24)(18) - (0.12)(35)$

$r \times F = I \cdot \alpha$

- 'F = ma'
- I = Moment of inertia

$$I = \Sigma mr^2$$

Rotation Example

A rigid bar with a mass M and length L is free to rotate about a frictionless hinge on a wall. The bar has a moment of inertia of $I = 1/3ML^2$ around the hinge and is released from rest from an initially horizontal position as shown. What is the angular acceleration of the bar when it makes a 30.0° angle to the vertical also as shown?

A .) $\frac{g}{3L}$

B.) $\frac{2g}{3L}$

C.) $\frac{g}{L}$

D.) $\frac{3g}{4L}$

E.) $\frac{3g}{2L}$

$I = \frac{1}{3}ML^2$ M L

$\alpha = ?$

$\tau = I \cdot \alpha$

$\tau = \frac{1}{3}ML^2 \cdot \alpha$

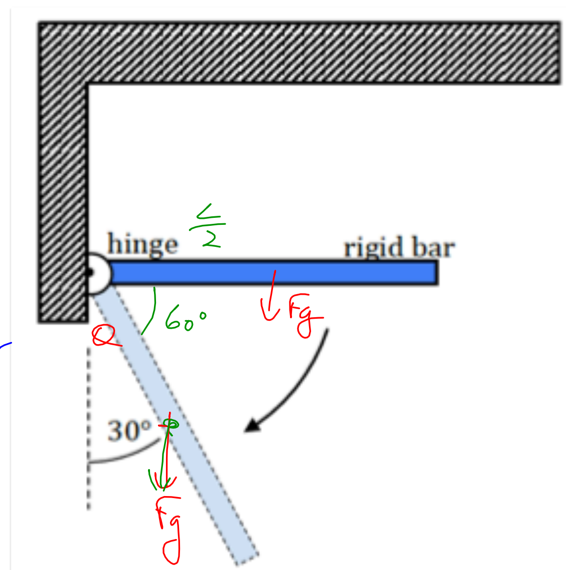
$rF\sin\theta = \frac{1}{3}ML^2 \cdot \alpha$

$\frac{L}{2} \cdot \cancel{M} g \sin 30^\circ = \frac{1}{3}\cancel{M}L^2 \cdot \alpha$

$\frac{\cancel{L}}{2} \cdot g \cdot \frac{1}{2} = \frac{1}{3} \cdot L^{\cancel{2}} \cdot \alpha$

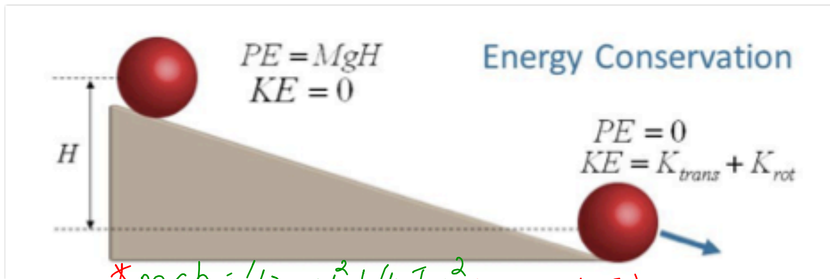
$\frac{1}{4} g = \frac{1}{3} L \cdot \alpha$

$\frac{3g}{4L} = \frac{L}{3}\alpha$



Rotation

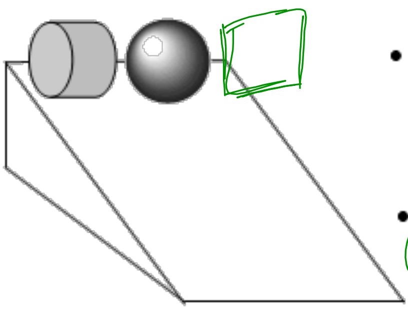
- Rotational Kinetic Energy: ½ $I\omega^2$



$* \; mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$ $\quad v = r\omega$

- Gravitational potential converted into translational & rotational KE

$\omega^2 = \frac{v^2}{r^2}$



- Smallest moment of inertia wins – easier to begin rotation

- Of a cylinder, solid sphere, and a box, which would win? Why?

Rotation

- Angular momentum – tendency of an object to continue rotation.
  - Conservation of angular momentum

$$L = I \cdot \omega$$

$$L_o = L_f = \text{constant}$$

- Alternate formulas:

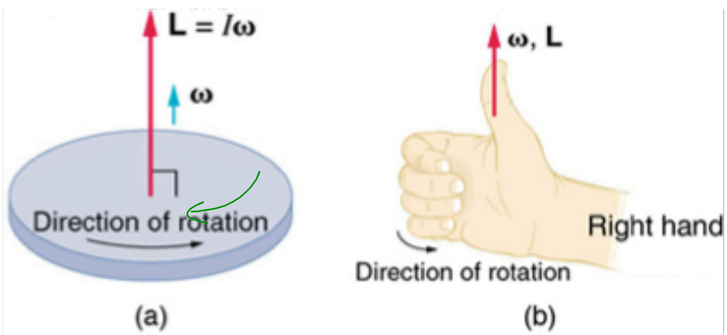
$$L = mrv$$

$$v = r\omega$$
$$\omega = \frac{v}{r}$$

$$L = I \cdot \omega = mr^2 \cdot \frac{v}{r}$$

$$\tau = \frac{\Delta L}{\Delta t} \rightarrow \text{Impulse-Momentum Theory}$$

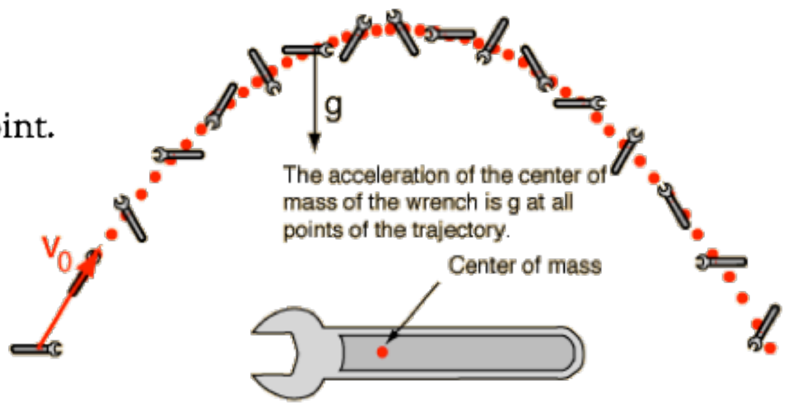
- **Direction of angular momentum and additional torque



*Center of Mass

- Average mass of an object condensed into a single point. Average location of mass

$$* \quad x_{CM} = \frac{\sum_i m_i x_i}{M} \qquad y_{CM} = \frac{\sum_i m_i y_i}{M}$$



A 100.0 g mass is placed at one end of a 100.0 g meter stick. If the meter stick is said to be balanced on a pivot point at the 20.0 cm mark as shown, how much mass must be at the 10.0 cm mark?